Krapivsky-Redner modification of nonlinear Barabási-Albert networks

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Abstract:

In growing Barabási-Albert (BA) networks, a new node randomly selects an existing target node and attaches to it randomly with a probability r proportional to the number k of neighbors already attached to the target node. Krapivsky and Redner use, also for different networks: "a new node randomly selects an existing target node, but attaches to a random neighbor of this target." In nonlinear BA networks, r is made proportional to $k^\alpha$ with $\alpha = 1$ for the standard BA case. We simulate here nonlinear Barabási-Albert-Krapivsky-Redner (BAKR) networks, where r is applied to the selection of the target, not to the selection of the target neighbor. We use undirected Barabási-Albert networks. For the maximum number kmax of neighbors we find little effect from $\alpha$, while the distribution n(k) of the number of neighbors has a normal power law and there is no gap or strong peak in the number of neighbors k(i). All this contradicts our earlier simulations without redirection.

Keywords: Barabasi Albert network, probability, number of neighbors, nodes.
Introduction:

Barabási Albert networks (BA) and their modifications have been studied for several years[1,2]. We studied the nonlinear Barabási-Albert network (NLBA) where a new node connects to a vertex having k neighbors with a probability r [3] proportional to $k^\alpha$ real; usually $\alpha = 1$. Each new node adds m new edges to the network, and we developed two versions from this model NLBA1 and NLBA2 [4,5]. We now study the modification of Krapivsky and Redner who use, also for different networks: “a new node randomly selects an existing target node, but attaches to a random neighbor of this target” [6,7].

Krapivsky-Redner networks use redirection as a fundamental network growth mechanism to determine how a new node n attaches to a growing network. For undirected networks, without a prescribed direction for each link, redirection is implemented as a new node n chooses from all already existing nodes a provisional target node at random, with probability $0 < r < 1$. Then a randomly selected neighbor of the target attaches to the ancestor n of the target. We now investigate nonlinear Barabási-Albert-Krapivsky-Redner (BAKR) networks, where $r$ is applied to the selection of the target, not to the selection of the target neighbor. We use undirected nonlinear Barabási-Albert networks.

Data and Simulation:

In our simulations we use the Fortran program of BAKR as in the appendix. In the simple version, it uses a rectangular array $\text{neighb}(\text{maxnb, max})$, where max = N + m is the network size and maxnb the maximum number of neighbors for a single node. With small networks, we start with a parameter maxnb = 1.1*maxtime*m, but in large networks, we use maxnb = 0.9*maxtime*m, since we must adjust the parameter maxnb with small N such that it is never smaller than the maximum $k_{\max}$ of the number of neighbors $k(i)$ over all node indices i. With maxnb=max*m one has the absolute maximum: each of the max new nodes n produces m new neighbor bonds. However, our first tests show that $k_{\max}$ is much smaller, and then one can save memory by using a smaller parameter maxnb. When we vary $\alpha$ between 0.1 and 1.9, with
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$m = 4$, and observe values of $k_{\text{max}}$, of the number of neighbors $k(i)$ of node $i$, and of the number $n(k)$ of nodes with $k$ neighbors, we notice no gaps, no strong peaks, and normal power law in the $n(k)$, and $k_{\text{max}}$ varies little with varying values of $\alpha$. We simulate different values of lattice size ($N=1000, 2000, 4000, 8000, 16000, 22000$) with $\alpha$ up to 0.1 with constant values of $m=4$. For $N=16000$ and 22000 nodes, we used Allocatable command of fortran90 because maxnb is large. We use a logarithmic scale at the y-axis ($k_{\text{max}}$) to let results become clearer, and show $k_{\text{max}}$ for small and large $N$ in figure (1).

Figure (1): $k_{\text{max}}$ versus $\alpha$ up to 0.1 for different size $N$
Figure (2) shows $k_{\max}$ versus $\alpha$ for larger values of $\alpha$ up to 1.9.

Now we fix $N=22000$, $m=4$, but change $\alpha$ from 0.1 to 1.9, and we get figure (3) which shows the number of $k(i)$ of neighbors versus index of nodes $i$. 
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Figure(3): number $k(i)$ of neighbors versus index of nodes $I$ with $m=4$ and $N=16000$ as example.

When we fix $\alpha$ at 0.5 and 1.5 with changing size $N$ from 1000 to 7000 at constant $m=2,4,8,16,32,64,128,256$. We get figure 4. It shows that $k_{\text{max}}$ increases roughly proportional size $N$ at fixed $\alpha$ and number $m$ of neighbors.
Figure(4): kmax versus size N versus at $\alpha$ 0.5 [m=8(*) m=4(+)] and $\alpha$ 1.5[m=4(empty squ.) m=8(x)].

We see that kmax is always much smaller than N, in particular there is no strong dependence of kmax on $\alpha$, while in [4] we found kmax to increase strongly up to close to the theoretical maximum N*m for $\alpha > 1.5$.

We simulate many values of m=2,4,8,16,32,64,128,256 as we can in our computers, for small $\alpha=0.5$ and with size = 1000, and we take nklog is the sum over all n(k) within an interval proportional to k, the n(k) vary roughly as 1/k**3, while the slope in our curves in figure(5) is close to -2.
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Figure (5): $n(k)$ versus $k$ versus $\alpha^{0.5}$ [m=2,4,8,16,32,64,128,256 from bottom to top with N=1000].

Finally, we plot figure 6 as $k_{max}$ versus different $m$ which shows us the relation between $k_{max}$ and $m$ which is: $k_{max}$ proportional directly to $m$. 

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Figure (6): kmax versus m at α = 0.5 with multiple of m

**Conclusion:**

We observed no gaps in k(i), no peaks in k(i), no strong variation of kmax versus α with the Krapivsky-Redner modification. We see that kmax is always much smaller than N, in particular far below the theoretical maximum N*m. All these results differ from our earlier simulations without redirection [3,4,5].

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