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The effect of Lattice Size on one-dimensional Majority-vote model on directed Small-World Networks

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الملخص

تأثير حجم الشبيكة على نموذج التصويت بالأغلبية أحادي البعد في شبكات عالم صغير موجهة

في هذا البحث، تم دراسة النموذج الفيزيائي- الاجتماعي في بعد واحد (D1) المسمى نموذج التصويت بالأغلبية (MVM) وخصائصه الفيزيائية الحرجة باستخدام شبكات العالم الصغير الموجهة بدقة (DSW). استخدم الباحثون طريقة مونت كارلو لدراسة MVM لأحجام مختلفة من الشبيكة (L) مع احتمال = 0.95، والذي يعتبر نموذج موجه بدقة مع الروابط غير المحلية. بالإضافة إلى تحديد نسبة الأس الحرجة، باستخدام النسب الأسية الحرجة ($\frac{\gamma}{\nu}$، $\frac{\beta}{\nu}$ ، $\frac{1}{\nu}$). حيث أظهرت النتائج أن MVM يقدم نسب أسية مطابقة لنموذج Ising أحادي البعد على شبكات العالم الصغير الموجهة، كما أظهرت تطابقاً مع نماذج غرينشتاين المعيارية (Grinstein Criterion models)، داخل تماثل أعلى وأسفل الشبكات العادية.

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Abstract

This research, studies the social-physical model in one-dimension (1D) which is known as Majority-Vote model (MVM) and its critical physical properties using the extremely directed small-world networks (DSW). The researchers used the Monte Carlo method to study the MVM for different lattice sizes (L) with probability=0.95, which is the case of the extremely targeted model with nonlocal links. In addition, the critical exponents' ratio are determined, by using critical exponent's ratios γ/ν, β/ν, and $1/\nu$. The results show that MVM presents identical exponents to the one-dimensional Ising model on directed Small-World networks.

Keywords: Ising model, Lattice Size, Probability, Majority-vote, Monte Carlo simulations.

1. Introduction

In 1985 Grinstein et al. [1] argued that models on regular lattices with up-down symmetry belong in the universality class Ising models[1]. This hypothesis has been confirmed for some non-equilibrium models on regular lattices [3,4,5,6,7,8,9].

Oliveira in 1992 [2] proposed the non-equilibrium model known as MVM, which defies the detailed balance. The update of the MVM follows a Markov sequence of stochastic dynamics with local rules and with up-down symmetry. In two dimensions, on a square lattice, the MVM presents a continuous phase transition with congruous critical exponents [2] of the Ising model [10]. Sousa and Brenda studied the Ising model and MVM on DSW random lattices [11,12], respectively. The exponents obtained identical results in both models and in agreement with the conjecture suggested by Grinstein et al. [1].

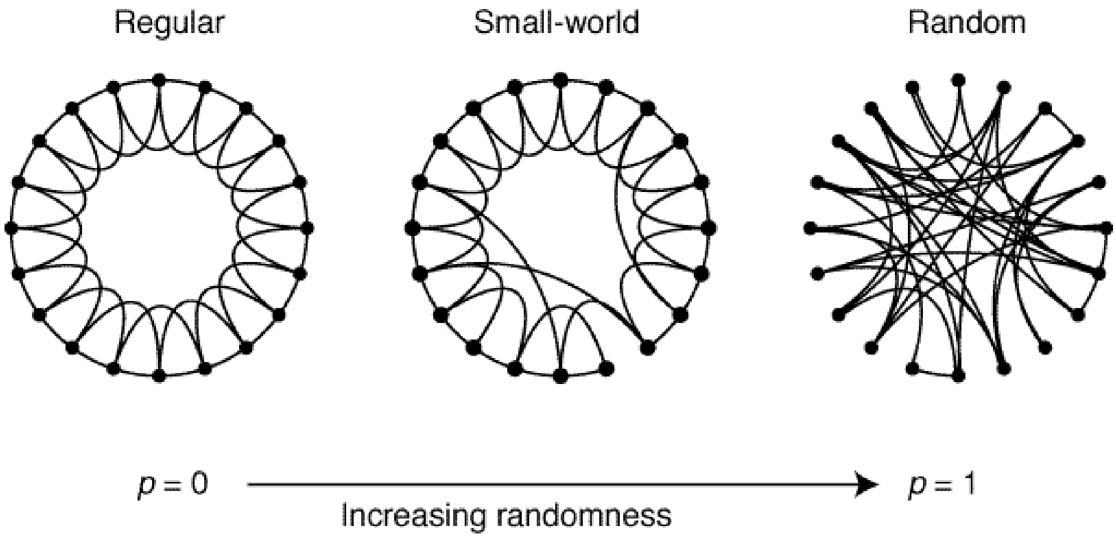

Figure 1.: Random networks with different values of probability (P=0, 0<P<1, P=1).

2. Model and simulations

The MVM dynamics contain a noise parameter q. For square lattice of four neighbors when q = 0; if three or four neighbors contradict with the central site, the centre flips; if one or none disagrees, the centre does not flip; if two neighbors correspond and the rest do not, the centre flips with probability 1/2. When q > 0 the centre may disobey to this majority rule. Therefore, we considered the non-equilibrium MVM with q noise on extremely DSW networks by a set "voters" or spins variables σ taking the values +1 or −1 located on every node or sites of a DSW network with L sites such as (5M,10M, 20M, 30M, 50M, 80M, and 120M) whereas M = 1000, where L is the length of a linear chain as in appendix(1).

The one-dimension of a small-world network was built from a regular network related to two nearest neighbors, connected to L nodes and J neighbors. In the DSW, each node is randomly reconnected with n edges with probability p. Using the values of probability between 0 and 1; when p = 0 for the network it is regular (received no long-range connection), in the case where the values of probability were 0 <p <1 the system is small world (short-range links), while when p = 1 random network (long-range

connections), as shown in figure 1.
For MVM on a network, a spin or 'voter' variable $\sigma_i = \pm 1$ were put at each node of the network. At every single step, the researchers tried to spin- flip a node. The flip is accepted with probability [1].

$$W_i = \frac{1}{2}[1-(1-2q)\sigma_i . S(\textstyle\sum_j \sigma_j)] \qquad (1)$$

where S(x) is the sign of x if $X \neq 0$, S(x) = 0, if x = 0. To calculate W_i the sum runs over the (J=2) nearest neighbours of spin i on the network.

By adding a long-range connection connecting to another site k with p=0.95, which is the case of the extremely targeted model with nonlocal links. This connection is the only one way, that is, the site k does not send back a connection to the site i. W_i means that with probability (1-q) the spin will adopt the same state as the majority of its neighbours. The control noise parameter q($0 \leq q \leq 1$) works like the temperature in the Ising model: the smaller the value of q, the greater the likelihood of parallel alignment with the local majority. These simulations were performed on different DSW network sizes and different values of L. For each L size quenched averages over the connectivity disorder are approximated by averaging over independent realizations. For each simulation, the researchers started with a stable configuration of spins. They ran 3×10^5 Monte Carlo steps (MCS) per spin with 2×10^5configurations discarded for the system to reach a steady-state using a random number generator.

To study the MVM, the research considered the molar magnetization m, as $m = \sum_i \sigma_i / L$, where σ_i is a spin or 'voter' variable at each node of the network, and L is the length of a linear chain, also from magnetization, it's possible to investigate other measures such as the average magnetization, susceptibility and the fourth-order Binder cumulant,

$$m = [\langle |m| \rangle]_{av} \ , \qquad (2)$$

$$\chi(q) = \frac{L}{T}\,[\langle m^2 \rangle - \langle m \rangle^2]_{av} \ , \qquad (3)$$

$$U_4(q) = 1 - \left[\frac{\langle m^4 \rangle}{3\langle |m| \rangle^2}\right]_{av} \tag{4}$$

The above equations < …. > stand for thermodynamic averages and $[\ldots]_{av}$ for averages over different realizations. To calculate the exponents of these models, the researchers apply the finite-size scaling (FSS) theory [10]. They expect, for large system sizes, an asymptotic FSS behaviour of the form:

$$m = L^{-\beta/v} f_m(x)[1 + \cdots], \tag{5}$$

$$\chi = L^{\gamma/v} f_\chi(x)[1 + \cdots], \tag{6}$$

where β and γ are the usual critical exponents, and $f_i(x)$ are FSS functions with

$$\chi = (q - q_c)L^{1/v} \tag{7}$$

being the scaling variable, where q_c is the critical noise parameter. The dots in the brackets [1+...] indicate corrections to scaling terms. The researchers calculated the error bars from the fluctuations among the different realizations. Therefore, from the size dependence of M and χ, they obtain the exponents' ratios β/v and γ/v , respectively. The susceptibility at its maximum also scales as $L^{\gamma/v}$.
Moreover, the value of q for which χ has a maximum, $q_c^{\chi max} = q_c(L)$ scales with the lattice size as:

$$q_c(L) = q_c + bL^{-1/v} \tag{8}$$

where b is a non-universal constant.
The correlation length exponent 1⁄v can be estimated from eq.(8).

3. Results and discussions

Figure (2) shows the system organization parameter, also called magnetization versus the noise parameter *q* for sizes L=5M, 10M, 20M, 30M, 50M, 80M, and 120M and rewiring probability *p=0.95*. From figure 2, it's noticed that this model illustrates a continuous phase transition and the decay behavior of magnetization agrees with magnetization universality. Moreover, the magnetization goes to zero at q≅0.7625.

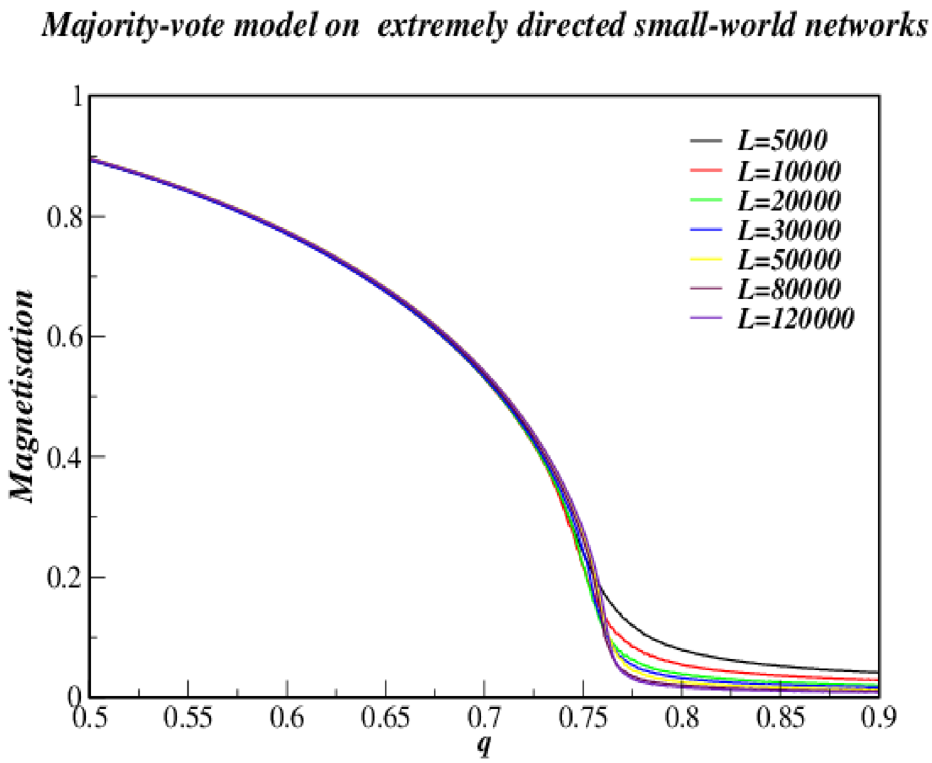

Figure 2. : Magnetization versus noise parameter q for sizes L=5M, 10M, 20M, 30M, 50M, 80M, and 120M with rewiring probability p=0.95.

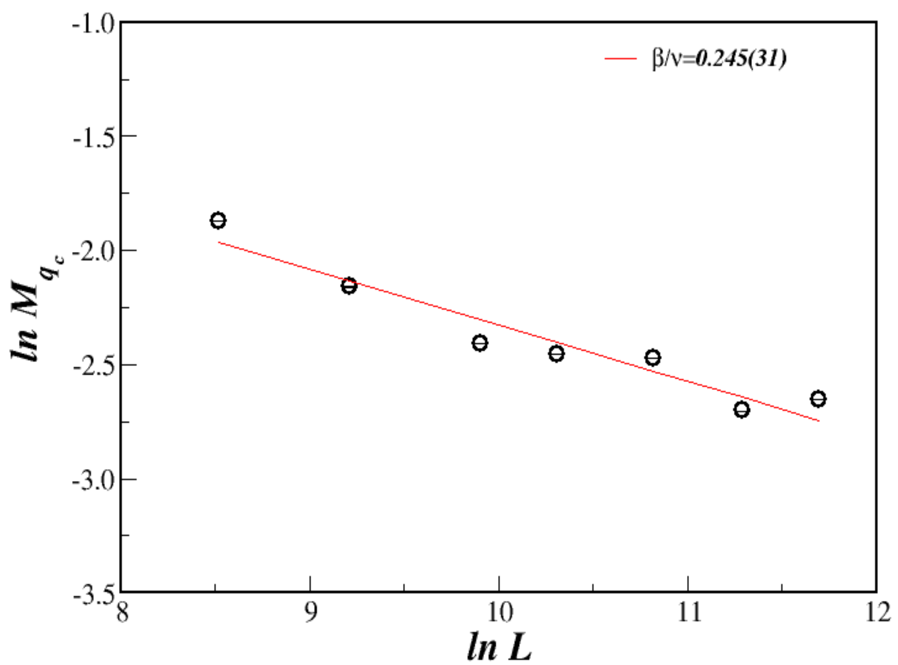

Figure 3. Logarithm of the magnetization at q_c as a function of the logarithm of lattice size L for rewiring probability p = 0.95.

The slope of the straight line in figure 3 can be obtained through linear regression.

By using eq. (5) and this slope we got the exponents' ratio $\boldsymbol{\beta/v = 0.245(31)}$.

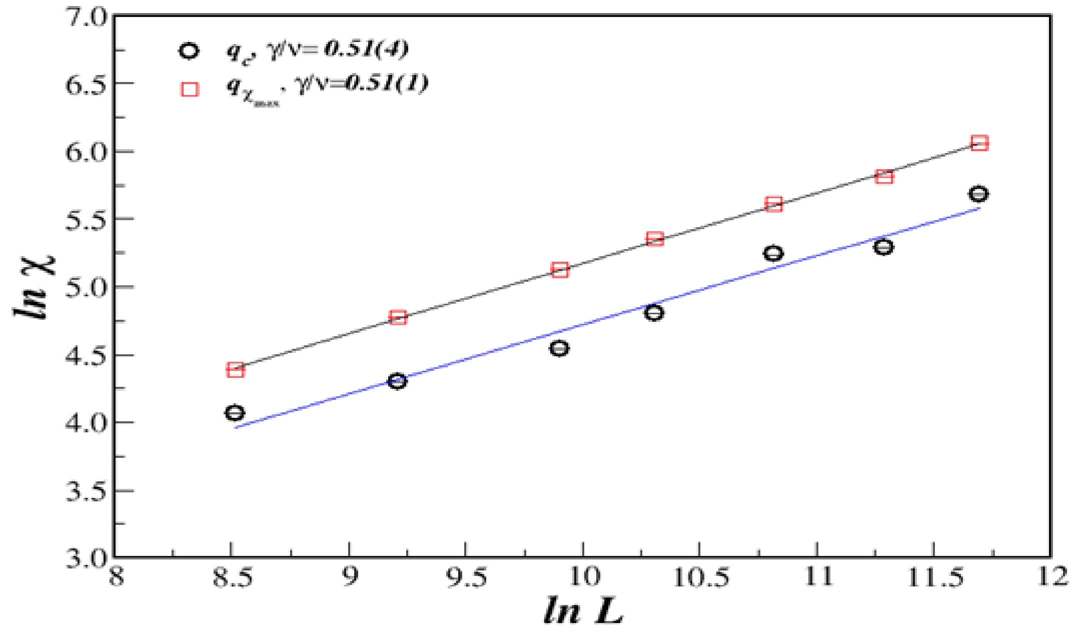

Figure 4. Logarithm of the susceptibility χ_{q_c} and χ_{max} versus Logarithm of lattice size L for rewiring probability p = 0.95.

In Figure 4, the logarithm of the fluctuation of magnetization also known as susceptibility χ at q_c and χ_{max} versus the logarithm of lattice size (L) is plotted . Through a linear regression of the straight lines in figure 4 and using eq. (6), the exponents' ratios $\Upsilon/v_{q_c} = 0.51(4)$, and $\gamma/v_{q_{max}} = 0.51(1)$ for rewiring probability p = 0.95 were obtained.

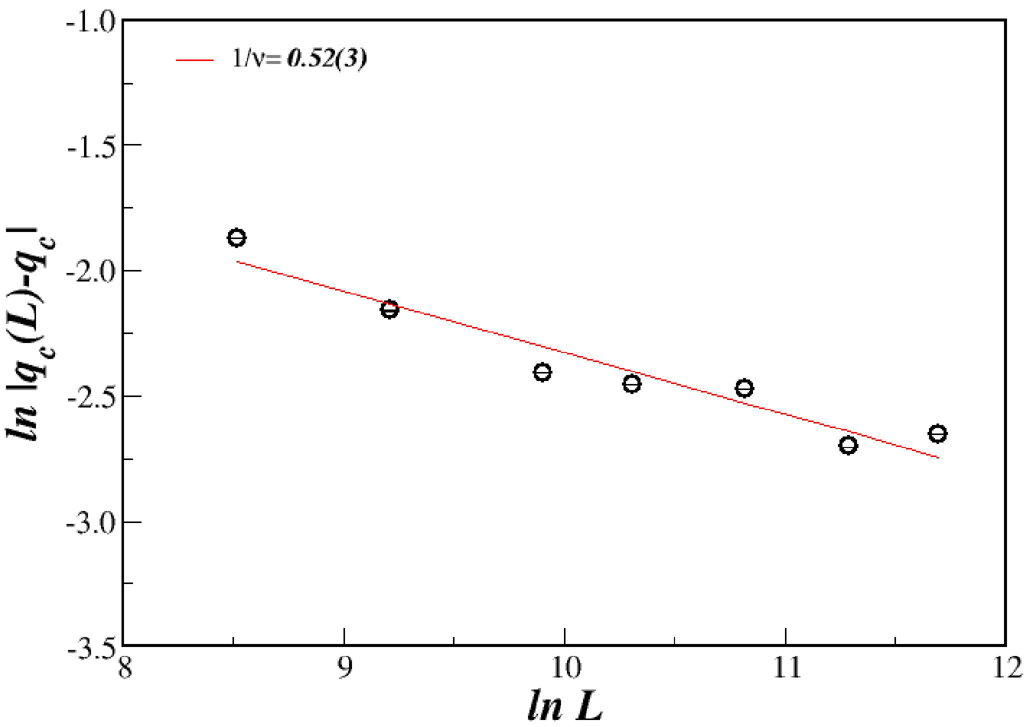

Figure 5. $\mathrm{Ln}[q_c(L) - q_c]$ versus ln L with p=0.95.

Figure 5 is obtained by calculating the values of $[q_c(L) - q_c]$, and plotting the logarithm of these values versus the logarithm of L and then from equation (7), the exponents' ratio $1/v = 0.52(3)$ is obtained.

Moreover, a plot of susceptibility versus q with probability(P=0.95) for different L sizes, L=5M, 10M, 20M, 30M, 50M, 80M, and 120M, (figure 6,) shows a shift in the peak for as the lattice size (L) increases, see table1:

Table (1): Peak values coordinates for different lattice size

L	**Vales on y-axis**	**Vales on x-axis**
5M	0.746	79.993
10M	0.748	123.418
20M	0.751	168.340
30M	0.755	208.770
50M	0.758	268.665
80M	0.760	336.048
120M	0.762	412.415

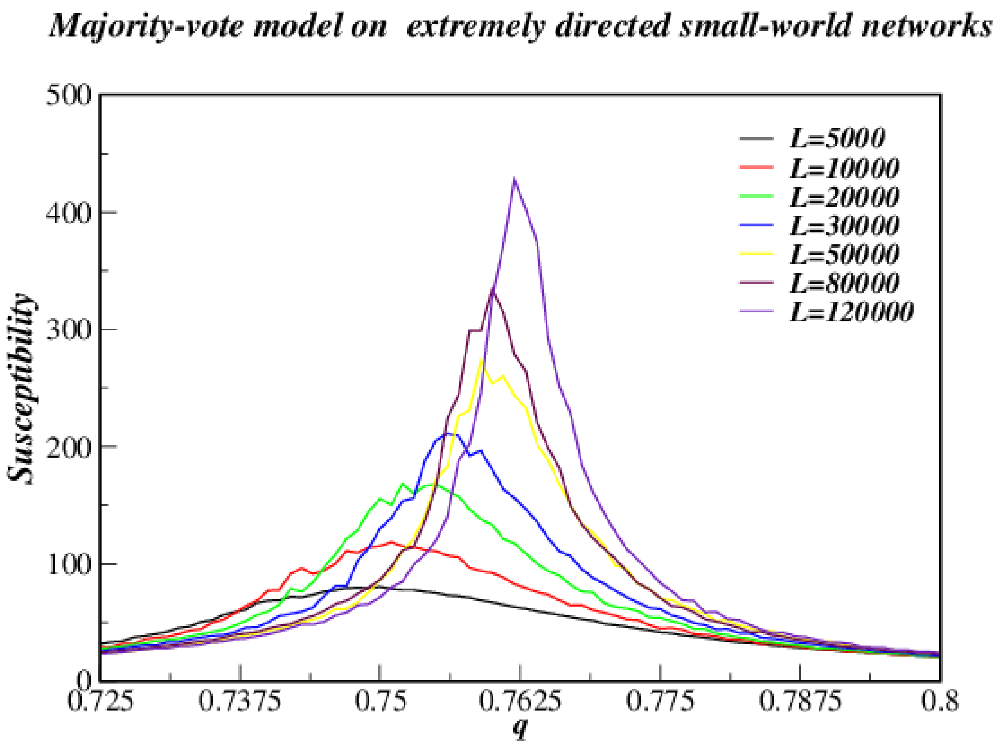

Figure 6. Susceptibility versus noise parameter q for sizes L=5M, 10M, 20M, 30M, 50M, 80M, and 120M with rewiring probability p=0.95.

Figure 7, demonstrates the fourth-order Binder cumulant (U4), versus the noise parameter *q* for different sizes L=5M, 10M, 20M, 30M, 50M, 80M, 120M, with probability p=0.95.

The shape of figures 5,6 and 7 show a continuous phase transition, in addition, all curves nearly intersect at q=0.7625, furthermore all curves start to decays at point 0.715 to 0.761.

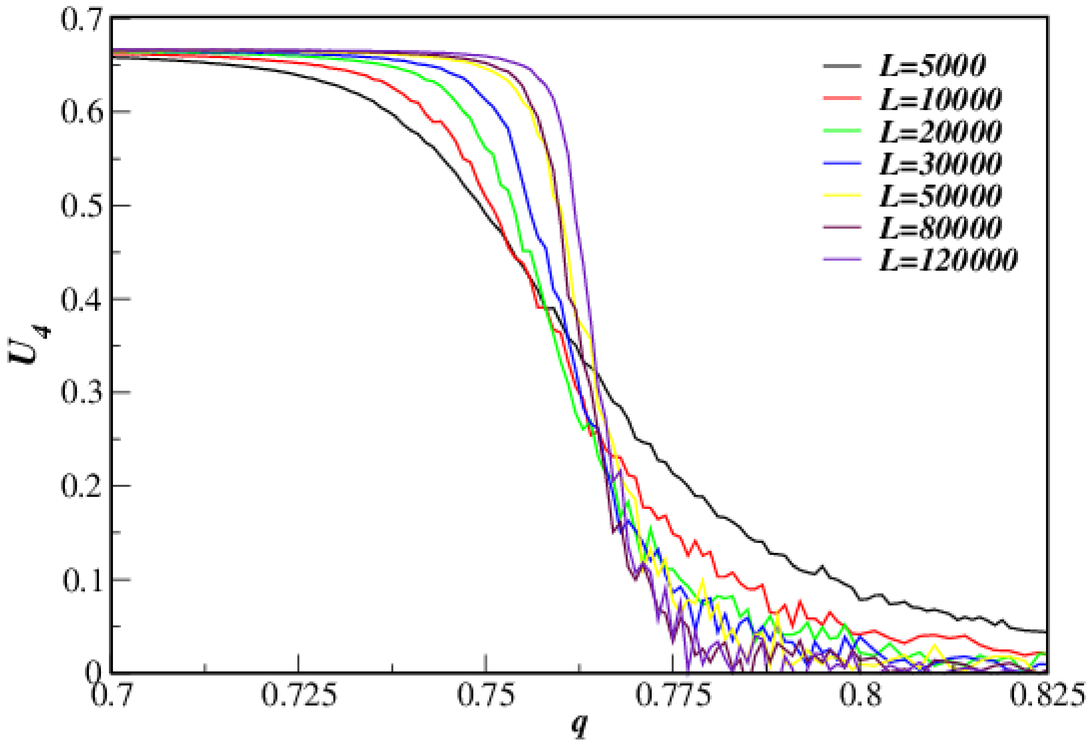

Figure 7. Fourth- order Binder Cumulante(U4), versus the noise parameter q for different sizes L=5M, 10M, 20M, 30M, 50M, 80M, 120M, with rewiring probability p=0.95

4. Conclusion

The researchers conclude that the Majority-Vote model on directed SmallWorld Networks (DSWN) in one dimension ferromagnetic, had a continuous phase transition, in addition, they estimated the exponents' ratios $\beta/v = 0.245(31)$, $\Upsilon/v_{q_c} = 0.51(4)$, and $1/v = 0.52(3)$, for p=0.95 .

From critical exponents' ratios obtained here for the non-equilibrium MVM model, it's observed that the critical exponents are identical to the equilibrium Ising model[13]. These results show that these models belong in the same universality class and are in accordance with Grinstein et al. [1].

5. References

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Appendix(1):

```
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
    parameter(idim=1,L=120000,L2=L**idim,Lmax = L2+2*(L**(idim-1)))
```

```
      parameter(k=3,nsamp=10, itmax=700, p=0.95d0 )
      character*40 FILE1,FILE2,FILE3,FILE4,FILE5,FILE6
c      real*8 ex,dt,T,beta,factor
    integer*8 ibm,ip,iex(-1:+1,-k:k)
    byte is
    logical ic(Lmax)
    dimension is(Lmax), neighb(Lmax,k),nk(Lmax)
    NREDE=1
      ip=2147483648.0d0*(4.d0*p-2.d0)*2147483648.0d0
      factor=(0.25d0/2147483648.0d0)/2147483648.0d0
    DO 20 IREDE=1,NREDE
     iseed=1
     OPEN(UNIT=60,FILE='mag-L120000_p095.dat')
     OPEN(UNIT=61,FILE='chi-L120000_p095.dat')
     OPEN(UNIT=62,FILE='um4-L120000_p095.dat')
       ibm=2*iseed-1
     anorma=dfloat(L2)
     T=0.40d0
c    boundary contourn
     Lp1=L**(idim-1)+1
     L2pL=L**(idim)+L**(idim-1)
     do 12 i=Lp1,L2pL
```

```
      neighb(i,1)=i-1
      neighb(i,2)=i+1
      ic(i)=.true.
      nk(i)=2    !CORRECTED !initialy all has 2 neighbors.
12        continue
      do 31 i=1,L2
      if(ic(i)) then
         j=3
         ibm=ibm*16807
      if(ibm.Lt.ip)then
40       ibm=ibm*16807
      new=1.d0+(0.5d0+factor*ibm)*L2
      i1=iabs(new-i)
      if(new.gt.L2.or.new.le.0.or.i1.le.1.or..not.ic(new))goto 40
         if(new.ne.neighb(i,j))then
      neighb(i,j)=new
      ic(new)=.false.
      nk(i)=nk(i)+1
      endif
c      write(*,*)i,j,new,Lp1,L2
         endif
      endif
```

```
31      continue
          do 8 it=1,itmax
          beta=1.d0/t
       dq=tanh(beta) ! Glauber trantion rate
C ******************************************
c         probability table
c******************************************
      do i=-1,1
      do j=-k,k
      ie=j*i
      isgn=0
      if(ie.gt.0) isgn=+1
      if(ie.lt.0) isgn=-1
      ex=0.5d0*(1.d0 - dq*isgn)
      iex(i,j)=2147483648.0d0*(4.d0*ex-2.d0)*2147483648.0d0
      enddo
      enddo
       mcstep=80000  !here is necessary
       itrelax=30000
       dt=0.002D0
      IF(T.GT.0.7d0.AND.T.LT.0.8d0)THEN
       dt=0.001d0
```

```
        mcstep=90000  !here is necessary
        itrelax=60000
        ENDIF
        rchi=0.d0
              rchi2=0.d0
        rbinder=0.d0
        rbinder2=0.d0
        avermag=0.d0
        avermag2=0.d0
              DO 30 isamp=1,nsamp
C   INITIAL CONFIGURATION(ISING MODEL) ORDENADA
C
C      ACUMULADORES
          avexmag=0.d0
          avexmag2=0.d0
                avexmag4=0.d0
          icount=0
c
C*************************************************
C       and initial configuration is(i)=1
       do 5 i=Lp1,L2pL
5       is(i)=1
```

```
      icount=0
      do 1 mc=1, mcstep
      do 6 j=1,L**(idim-1)
       nk(j)=nk(j+L2)
       is(j)=is(j+L2)
       ic(j)=ic(j+L2)
       ic(j+L2pL)=ic(j+L**(idim-1))
       nk(j+L2pL)=nk(j+L**(idim-1))
6       is(j+L2pL)=is(j+L**(idim-1))
      do 3 i=Lp1,L2pL
        isi=is(i)
        ie=0
        do 4 j=1,nk(i)
4         ie=ie+is(neighb(i,j))
        ibm=ibm*16807
3        if(ibm.lt.iex(isi,ie)) is(i)=-isi
        if(mc.GT.itrelax)then
        icount=icount+1
         mag=0
         do 10 i=1,L2
         mag=mag+is(i)
10            continue
```

```
c       call flush(6)
        avexmag =avexmag+abs(1.d0*mag/anorma)
        avexmag2=avexmag2+(1.d0*mag/anorma)**2
        avexmag4=avexmag4+(1.d0*mag/anorma)**4
        endif
c       PRINT *,mc, to ,
1       continue
        rcount=dfloat(icount)
        amag= avexmag/rcount
        amag2=avexmag2/rcount
        amag4=avexmag4/rcount
            avermag=avermag+amag
        avermag2=avermag2+amag**2
        xchi=L2*(amag2-amag**2)*beta
c       answer functions
        rchi=rchi+xchi
        rchi2=rchi2+xchi**2
        fat0  =3.d0*amag2**2
        fat1  =1.d0-amag4/fat0
        rbinder=rbinder+fat1
        rbinder2=rbinder2+fat1**2
30      CONTINUE
```

```
      rsamp=dfloat(nsamp)
      deno=rsamp*rcount-1.d0
      po=avermag/rsamp
      po2=avermag2/rsamp
      pchi=rchi/rsamp
      pchi2=rchi2/rsamp
      pul =rbinder/rsamp
      pul2=rbinder2/rsamp
      erpo=(po2-po**2)/deno
      erpo=ABS(erpo)
      erpo=SQRT(erpo)
      erpc=(pchi2-pchi**2)/deno
      erpc=ABS(erpc)
      erpc=SQRT(erpc)
      erpu=(pul2-pul**2)/deno
      erpu=ABS(erpu)
      erpu=SQRT(erpu)
      WRITE(60,*)T,po,erpo
      WRITE(61,*)T,pchi,erpc
      WRITE(62,*)T,pul,erpu
c      WRITE(*,*)T,po,PCHI,pe
      xpn=T
```

```
      if(xpn.ge.0.99)goto 20
      t=t+DT
c       print *, t,po,pe
8       continue
20       continue
      stop
      end
```