In this paper we study urban segregation of two different communities A and B, rich and poor, distributed randomly on finite samples, to check cheap and expensive residences. For this purpose we avoid the complications of the Schelling model which are not necessary and instead we use the Ising model on 500 × 500 square lattices, which gives similar results, with random magnetic field at lower and higher temperatures ($k_BT/J = 2.0, 99.0$) in finite times equal to 40, 400, 4000 and 40 000. This random-field Ising magnet is a suitable model, where each site of the square lattice carries a magnetic field $\pm h$ which is randomly up (expensive) or down (cheap). The resulting addition to the energy prefers up-spins on the expensive and down-spins on the cheap sites. Our simulations were carried out using a 50-line FORTRAN program. We present at a lower temperature (2.0) a time series of pictures, separating growing from non-growing domains. A small random field ($h = \pm 0.1$) allows for large domains, while a large random field ($h = \pm 0.9$) allows only small clusters. At higher temperature (99.0) we could not obtain growing domains.

**Keywords:** Opinion dynamics; sociophysics; random field Ising model; Schelling model.

1. Introduction

The Schelling model\(^1\) is a complicated version of a square-lattice Ising model at zero temperature, to explain urban segregation with two groups A and B, based on the neighbour preference of the residents, without external reasons. His version leads to small clusters\(^2,3\) but not to large domains as for blacks and whites in Harlem, New York City.\(^4\) Removing and replacing some people randomly\(^5\) helps, and one also gets “infinitely” large domains if happy (unhappy) people move to other places where they are happy (unhappy).\(^2,6\)

\(^5\)Presently at Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, 01187 Dresden, Germany.
Schelling\textsuperscript{1} asked whether the racial segregation in American cities can emerge from intrinsic behaviour of the individual people, instead of or in addition to extrinsic reasons like discrimination, rent differences, etc. In particular, can “black” ghettos in the predominantly “white” USA arise just because people prefer to have neighbours of their own group over neighbours from the other group? In many other countries we find many other types of residential segregation, based on religion, ethnicity, ... In physics, such a process is easily simulated through the two-dimensional Ising model.

Temperature here can have two meanings: tolerance and noise.\textsuperscript{7} Tolerance means that for high $T$ one is willing to live among neighbours from a different group, and low $T$ means that he strongly prefers to live among neighbours of his own group. The alternative interpretation is $T = \text{noise}$; $T$ then measures all those facts of life outside the model which force people to move to another residence even though they like their old residence better. While then the domains in the Schelling model at positive temperature seem to grow towards infinity,\textsuperscript{3} it is simpler to achieve the same aim in the well-known two-dimensional Ising model, with or without conserved number of people within each group.

Some researchers\textsuperscript{8,9} implemented a suggestion of Weidlich\textsuperscript{10} that people slowly learn to live together with neighbours from the other group. This learning means that the parameter $T$ (= temperature or tolerance) no longer is kept constant but slowly increases. An Ising model in Ref. 8 showed how an initial large domain dissolves if the temperature is slowly increased from below to above $T_c$. More realistically, for five (instead of only two) different groups in a modified 5-state Potts model,\textsuperscript{9} increased $T$ from low to high values and showed that with a slow increase one has appreciable domain formation during intermediate times, while with a fast increase this segregation is mostly avoided.

In the Schelling–Ising model\textsuperscript{1} of urban segregation, all lattice sites are equivalent. In reality, some houses are cheap and others are expensive.\textsuperscript{12} And usually of two groups in a population, one is poorer than the other. Our model uses two groups A and B of people, distributed on a square lattice, with the richer group A means up spins (+1), while the poorer group B means down spin (−1), and assumes that everybody prefers to be surrounded by lattice neighbours from the same group instead of from the other group. Flipping a spin then means that a person from one group leaves town and is replaced by a person from the other group, as simulated already in Ref. 5.

If there are whole neighbourhoods of expensive and cheap housing, then these housing conditions enforce a segregation of rich from poor, and this segregation does not emerge in a self-organized way. The more interesting case allows for self-organization of domains by assuming that each residence randomly is either expensive or cheap, with no spatial correlations in the prices.

In this paper we use the random-field Ising model to distinguish between cheap and expensive residences at one $T$. We present simulation at lower and higher
temperatures for different finite times. For such purpose we give a complete 50-line FORTRAN program. Parts of the text and program were taken from Refs. 13 and 11.

2. Random-Field Ising Model

In the Ising model, two neighbouring spins have, due to their interaction \(-JS_iS_k\), a higher probability to belong to the same group than to belong to the two different groups. If the difference between these two probabilities is large enough, \(T < T_c\), domain sizes can grow to infinity in an infinite lattice, while only small clusters are formed for smaller differences in the probabilities, \(T > T_c\). That these probabilities, controlled through \(-J/k_BT\), lead to these different regimes, separated by a sharp phase transition at \(T = T_c\), is not obvious from the definition of the interaction \(-JS_iS_k\), took physicists many years to find, and is typical of complex systems. The earlier standard Ising model gives results similar to the properly modified Schelling model.\(^5\,2,3\)

The magnetic field is considered to refer to the price of residence: people living in expensive residences are presented by positive magnetic field \(h > 0\), or in cheap residences presented by negative magnetic field \(h < 0\). Group A prefers to go to the expensive residences and group B to the cheap residences. This can be simulated by a random magnetic field which is \(+h\) on half of the places (attracting people of group A = up-spins) and is \(-h\) on the other half of the places (attracting group B = down-spins). The signs of the field are distributed randomly, and the model is called the random-field Ising model. This random field Ising model has a long history, and the asymptotic behavior for infinite system and infinite times is known. But for sociophysics we need finite systems and finite times and we use \(500 \times 500\) lattices and hundreds of time steps only. We simulate to what extent the two groups segregate.

In the spin 1/2 Ising model with Glauber kinetics of the square lattice, we interpret the two spin orientations as representing two groups of people. Nearest neighbours are coupled ferromagnetically, i.e., people prefer to be surrounded by others of the same group and not of the other group. Starting with random initial distribution of zero magnetization (= number of one group minus number of the other group), we check if “infinitely” large domain is formed. It is well known that they do so in zero magnetic field for \(0 < T < T_c\) where \(T_c = 2.269\) is the critical temperature in unit of the interaction energy. In this Ising model at finite \(T\), each pair \((i,k)\) of nearest neighbours produces an energy \(-JS_iS_k\) with some proportionality constant \(J\). The total energy \(E\) (= total unhappiness) is the sum of this pair energy over all neighbour pairs of the lattice. In statistical physics, different distributions of the spins \(S_i\) are realized with a probability proportional to \(\exp(-E/k_BT)\) where \(T\) is the absolute temperature and \(k_B\) the Boltzmann constant. There is no need to worry about values for \(T, k_B, J\) since the only relevant quantity is the ratio \(k_BT/J\), taken as 2 and 99.0 in our simulations. The Glauber kinetics is simulated
on the computer by flipping a spin only if a random number between 0 and 1 is smaller than the probability \( \exp(-\Delta E/k_B T)/[1 + \exp(-\Delta E/k_B T)] \), where \( \Delta E \) is the energy change produced by this spin flip. In addition we use a random field \( \pm h \); the probabilities proportional to \( \exp \left( -\text{Energy}/k_B T \right) \) now depend also on the (local) field by a factor \( \exp(\text{Field}/k_B T) \). The Fortran program used in this study contains 50 lines, takes a few seconds, and is presented below:

```fortran
parameter(L=500,Lmax=(L+2)*L,L2=L*L)
dimension is(Lmax),iex(9),iex1(9),iex2(9),h(Lmax)
byte is
data T,max,h0,ibm/2.0,400000,0.90,1/
print *,# L,max,ibm,t,h0',L,max,ibm,t,h0
Lp1=L+1
L2pL=L2+L
do 1 i=1,Lmax
is(i)=-1
ibm=ibm*16807
1 if(ibm.gt.0) is(i)=1
do 10 i=1,Lmax
h(i)=-h0
ibm=ibm*16807
10 if(ibm.gt.0) h(i)=h0
do 2 ie=1,9
ex1=exp(-2*((ie-5+h0)/T))
iex1(ie)=(2.0*ex1/(1.0+ex1) - 1.0)*2147483647
do 20 ie=1,9
ex2=exp(-2*((ie-5-h0)/T))
iex2(ie)=(2.0*ex2/(1.0+ex2) - 1.0)*2147483647
print *, '#iex1', iex1,iex2
ibm=2*ibm+1
do 3 mc=1,max
do 4 i=Lp1,L2pL
ie=5+IS(I)*is(i-1)+is(i+1)+is(i-L)+is(i+L)
ibm=ibm*16807
if(h(i)*is(i).lt.0) then
if(ibm.lt.iex2(ie)) is(i)=-is(i)
else
if(ibm.lt.iex1(ie)) is(i)=-is(i)
endif
4 continue
mag1=0
mag2=0
```
Various versions between Ising and Schelling models give about the same results. We therefore use in the present work the simple random-field two-dimensional Ising model with Glauber dynamics instead of the complicated Schelling model. In this study, initially we carry out our simulation at higher temperature of $T = 99.0$, with size of square lattice of $500 \times 500$, at time $= 400$, and vary the value of the field from 1.0 to 300.0 to check that our program agrees with the exact result for non-interacting spins. No large domains were noticed at high temperature of 99.0. It was also demonstrated that there are no growing domains obtained at high field. We then tested the growing of domains of groups in small random field of $0$.1, and low temperature of 2.0 at different times (40, 400, 4000, 40000) and the results of this simulation are presented in Figs. 1(a)–1(d).

The black domains in Fig. 1 refer to rich people while the white domains refer to poor people. We see in Fig. 1 from top to bottom a continuous growth of the domain size; at $t = 40000$ we have only one domain on top and one at the bottom. In addition there are always small black clusters in the white domains and small white clusters in the black domains. They would also occur on the standard Ising model at zero field and positive temperatures: Some spins are accidentally overturned because of thermal fluctuations.

We then repeat the simulation of the Ising model with the same above parameters used in Fig. 1 but in large random fields $0.9$ and we get Figs. 2(a)–2(d).

It can be noticed from Fig. 2 that there is strong preference of the poor to live in cheap houses and of the rich to live in expensive houses. And since cheap and expensive houses are distributed without domains (no rich quarters) the formation of large domains is prevented by the prices. The smaller in the price differences, makes the domains larger.
Fig. 1. Ising model simulation configurations after different times (a) 40, (b) 400, (c) 4000 and (d) 40000 per site on 500 × 500 square lattice at $k_B T/J = 2$ in a small random field of 0.1.
Urban Segregation with Cheap and Expensive Residences

Fig. 1 (Continued).

Fig. 2. Ising model simulation configurations after different times (a) 40, (b) 400, (c) 4000 and (d) 40,000 per site on 500 × 500 square lattice at $k_B T/J = 2$ in a large random field of 0.9.
3. Conclusion

We assumed that residences are either cheap or expensive, randomly distributed over the square lattice, and that two groups of people, rich and poor, make up the population. We found that for small fields after a long time the domains are larger than those of large fields in this random-field Ising model of urban segregation. Housing price differences do not prevent segregation if they are not very large.

Acknowledgments

The Authors would like to thank D. Stauffer, Institute for theoretical Physics, Cologne University for his valuable suggestions and constructive advice.
References