MONTE CARLO SIMULATION OF ISING MODEL
ON DIRECTED BARABASI-ALBERT NETWORK

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The existence of spontaneous magnetization of Ising spins on directed Barabasi-Albert networks is investigated with seven neighbors, by using Monte Carlo simulations. In large systems, we see the magnetization for different temperatures \( T \) to decay after a characteristic time \( \tau(T) \), which is extrapolated to diverge at zero temperature.

Keywords: Monte Carlo simulations; directed Barabasi-Albert networks; magnetization; Fortran program.

1. Introduction

The Ising magnet is, since decades, a standard tool of computational physics.\(^1\) We apply it here to scale-free networks,\(^2\) where previous simulations\(^3\) indicated a Curie temperature increasing logarithmically with increasing system size \( N \). In contrast to that work, we use here directed\(^4\) as opposed to undirected networks and then apply the standard Glauber kinetic Ising model\(^1\) to the fixed network.

2. Directed Barabasi–Albert Network

Putting Ising spins onto the sites (vertices, nodes) of a network, we simulate our Ising magnetic model on directed Barabasi–Albert networks. The Barabasi–Albert network is grown such that the probability of a new site, to be connected to one of the already existing sites, is proportional to the number of previous connections to this already existing site: the rich get richer. In this way, each new site selects exactly \( m \) old sites as neighbors.
Then, each spin is influenced by the fixed number $m$ of neighbors which it had selected when joining the network. It is not influenced by other spins which selected it as neighbor after it joined the network.

The Barabasi–Albert network is simulated by a Fortran program calculating the neighbors:

```fortran
parameter( nsites=500000, m=7, iseed=3, maxmax=20000,
1 max=nsites+m, length=1+2*m*nsites+2*m*m , T=1.0)
integer*8 ibm, iex
dimension list(length), is(max), iex(2*m+1), neighb(max,m)
ibm=iseed-1
factor=(0.25d0/2147483648.0d0)/21474836484.0d0
do 7 i=1,m
  do 7 nn=1,m
    neighb(i,nn)=nn
  7 list((i-1)*m+nn)=nn
L=m*m
c All m initial sites are connected
do 1 i=m+1,max
  do 2 new=1,m
    ibm=ibm*16807
    j=1+(ibm*factor+0.5)*L
    if(j.le.0.or.j.gt.L) goto 4
    j=list(j)
    list(L+new)=j
go to 4
  2 neighb(i,new)=j
  1 L=L+2*m
c print *,ibm,neigh
c end of network and neighborhood construction
```

At each step, a new spin is added which builds $m$ new connections `neighb`, randomly to already existing spins. The probability for an existing spin to be chosen as neighbor is proportional to the number of its neighbors, with the help of the Kertesz list.

### 3. Ising Magnet Using Monte Carlo Simulations

First, we initialize a directed Barabasi–Albert network with $m$ neighbors (all $m$ initial spins are connected with each other and themselves), here $m = 7$. We put an Ising spins onto every site, with all spins up, because we test here for ferromagnetism. Then, with the standard Glauber (heat bath) Monte Carlo algorithm spins, we search for thermal equilibrium at positive temperature all temperatures
are given in units of coupling constant over Boltzmann constant and change only between 0.5 and 1.0.

After putting all spins on the network, we go through the whole network and use the Monte Carlo step (MCS) on every spin; we say that we make one MCS per spin at each time step. Each spin is influenced by its exactly $m$ neighbors. We calculate the magnetization versus the number of time steps, with the same number of neighbors $m$ and different temperatures $T$.

Initially, we start with $T = 1.0$, and a number of spins equal to 500000, and time up to 20000. Then, we change the temperatures from 1.0 to lower values, for three samples with three iseed random numbers.

So, we can draw a graph of magnetization versus time for different temperatures to see how the magnetization changes, Fig. 1.

Now, we compare these graphs with each other by two ways:

1. We determine the time $\tau_1$ after which the magnetization has decayed to $3/4$ of its initial value (here 375000). This is done by plotting all results of different temperatures in one graph, we can draw the horizontal line “magnetization equal 375000”; then, the nearly vertical lines cross the horizontal line at the time $\tau_1$. So, we get different values of $\tau_1$ for different temperatures. Then, we draw the graph $1/\log_{10}(\tau_1)$ versus temperature as seen in Fig. 2.

Fig. 1. Magnetization versus MCS per spin, for $N = 500000$, time up to 20000, $m = 7$, iseed = 1, 3, 5 for different temperatures.
Reciprocal decadic logarithm of the two characteristic times, versus temperature

Fig. 2. $1/\log_{10}(\tau)$ versus temperature for $N = 500\,000$, time up to 20,000, $m = 7$, $\text{is}eed = 1, 3, 5$, three sample. The + signs refer to $\tau_1$, the × signs to $\tau_2$. The curves are parabolas corresponding to an asymptotic Arrhenius law $\tau \propto \exp(8.3/T)$.

(2) Alternatively, we define a $\tau_2(T)$ such that the magnetization curves $M(t, T)$, plotted as a function of the scaled time $t/\tau_2(T)$, agree with those for the reference temperature $T = 1.0$ where $\tau_2 = 1$. Thus, we get values of $\tau_2$ of each sample, changing with different temperatures. Then, we take the decadic logarithm of the average $\tau_2$ to draw $1/\log(\tau_2)$ versus temperature as shown also in Fig. 2.

4. Conclusion

We see that all figures agree with the modified Arrhenius law:

$$\frac{1}{\ln(\tau)} = 0.12 \cdot T + \text{const} \cdot T^2,$$

meaning that for each positive temperature, there is a finite relaxation time after which the initial magnetization decays towards zero: similar to the one-dimensional Ising model, there is no ferromagnetism on this directed Barabasi–Albert network.

Note Added in Proof

After a programming error was corrected, the curves of magnetization versus time changed qualitatively, but the conclusions remain. Corrected figures are seen on cond-mat/0411055 at www.arXiv.org.
Monte Carlo Simulation of Ising Model on Directed Barabasi–Albert Network

References